Tailing Futures Hedges/Tailing Spreads

by

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Abstract

An *untailed* hedge ignores the difference between the time futures gains or losses are realized and the time the price effects on the associated cash market exposures are realized. A tailed hedge, on the other hand, takes these timing considerations into consideration. Put another way, an untailed hedge ignores the effects of financing costs or investment returns associated with daily variation margin settlements of futures contracts; a tailed hedge these effects.

While tailed hedges should be recognized as more perfect from an economic perspective, untailed hedges have the advantage of offering the *appearance* of a better offset from an accounting point of view when deferral accounting methods are employed. Moreover, maintaining a correctly tailed hedge position requires an

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ongoing adjustment of the hedge position, while untailed hedges need no analogous adjustments.

This article also treats the concept of a tail in the context of spread trading. Here, the use of a tail allows the trader to take positions that reflect a judgment about expected changes in spread *yields* (i.e., the ratio of the two respective futures prices) as opposed to changes in spread prices (i.e., price differences), per se.

An important practical consideration has to do with size of positions -- whether in connection with hedging or spreading. When few futures contracts are desired and/or when exposure value dates are within fairly short-term horizons, the differences between tailed and untailed positions could easily be lost in rounding to the nearest whole number of contracts.

For significant institutional market users, however, the use of a tail offers the capacity to realize expected outcomes with greater precision.

“Tailing” in futures markets tends to be used in connection with two different applications. In hedge transactions, a tail offsets the incremental gains or losses from the interest associated with investment or financing of variation margin flows. In the context of spread trading, on the other hand, a tail insulates the position from the effects of changing spot market prices, so that dollar gains or losses follow only from changes in *yield* spreads. This article examines these two distinct applications. It explores when such tail positions are appropriate and how to determine their proper magnitudes.

I. Tailed Hedges

Those who use futures contracts for hedging should clearly understand the objective: A futures contract serves as a price-fixing mechanism. If properly designed and implemented, hedge profits will offset the loss from an adverse price move; in like fashion, hedge losses will also eliminate the effects of a favorable price change. Ultimately, the success of any hedge program rests on the implementation of a correctly sized futures position.

In concept, calculating the right size of a hedge is straightforward. To start, one needs to measure the effect of an instantaneous price perturbation on the underlying exposure. Then recognizing that the same price shock will generate a variation settlement effect on some associated futures contract, the proper size of the hedge is found by dividing the former effect by the latter. Complicating this calculation, however, is the issue of timing.

For instance, suppose in one case that the perturbation fosters an immediate $500 price effect on some exposure and a $25 effect on the associated futures contract. The correct hedge would be twenty futures contracts. In a second case, assume the same $500 exposure effect, but assume that it will only be realized some months, or even years from now. As the futures contract settles on a daily basis, a proper hedge should cover the *present value* of $500, which would clearly require *fewer than* twenty contracts.

Importantly, the correct number of contracts for this latter case will tend to increase as the passage of time erodes the difference between present values and future values. Ultimately, by the time the hedge value date is reached, the discounted present value will converge to the $500 amount. Thus, over time the required hedge will gradually rise to twenty contracts. This second case is an example of a *tailed hedge*, where the tail is the number of contracts needed to adjust for this present valuing effect.

In some situations, the appropriateness of tailing a hedge is obvious. Take, for instance, the objective of locking in the rate in advance of taking down a LIBOR-based loan. The effect of an interest rate change on this exposure is realized at the interest payment dates for the loan, say, three months following each rate-setting date. A futures hedge, on the other hand, generates immediate gains or losses each day as rates vary, from the time the hedge is implemented through the rate-setting date when the hedge is offset. Unambiguously, a tailed hedge is the proper economic solution to minimize risk.

In other cases, whether to tail or not may be less clear. For example, consider someone seeking to hedge or replicate a stock portfolio with stock index futures. Because the price effects on the portfolio and those of the hedge occur coincidentally, it might seem appropriate to use an untailed hedge. In fact, this intuition is not correct.

The solution to this problem requires manipulating the following system of equations

$$\begin{cases} F = \text{Index} \times (1+r)^n - \text{DIV} \\ \dfrac{\Delta E}{E} = \dfrac{\Delta \text{Index}}{\text{Index}} \\ \Delta E = H \times 500 \times \Delta F \end{cases}$$

Equation 1, 2, and 3

where F = the futures price;
Index = the stock index (spot) price;
r = the interest rate;
n = the number of (fractional) compounding periods to the futures value date;
DIV = dividend distributions (inclusive of reinvestment effects);
E = exposure to be hedged;
H = number of contracts to hedge for an instantaneous effect; and
500 = the multiplier dictated by the design of the futures contract.

Equation (1) is a statement of the "fair value" or the theoretical price of the futures contract. Equation (2) simply reflects the fact that any instantaneous percentage change of the portfolio value will be equal to the percentage change in the index value.[1] Equation (3) shows that the price effect of the futures contract offsetts the change in the portfolio value. Solving the system for H yields:[2]

$$H = \frac{E}{\text{Index} \times 500 \times (1+r)^n}$$

Equation 4

It should be clear that $\frac{1}{(1+r)^n}$ is the relevant present value factor for discounting from a forward date equal to the futures value date. Therefore, it turns out that H is, in fact, a tailed hedge.

To further demonstrate that H is a tailed hedge, consider a hedge with a time horizon equal to the value date of the associated futures contract. This hedge solution H* would be found as follows:

$$H^* = \frac{E}{\text{Index} \times 500}$$

Equation 5

This hedge, in effect, transforms the equity exposure into a money market yield, returning income made up of dividends (inclusive of associated reinvestment income) and income from the contract’s basis convergence. If the hedge is held to the contract expiration (i.e., no basis risk is involved), the resulting money market yield will be predetermined.

This outcome is demonstrated in Exhibit 1. That is, given Equation (5) for hedge calculation, the targeted money market yield is realized (5.20%, in this case), whether the stock market rises or falls. Clearly, any other hedge would generate different results, depending on the direction and size of the underlying price changes.

It should be noted that this conclusion requires two caveats: 1) All assumptions concerning dividends, the portfolio beta, and perfect convergence between spot and futures prices have to be realized; and (2) the outcome ignores all variation margin funding or investing effects. While nothing can be done to hedge against the first set of assumptions not being met, a tail can be employed to compensate for the variation margin effects.

Another way to look at the issue is the following: Ignoring the incremental income effects from investing variation margin gains (or borrowing to cover variation margin losses), we want the hedge to generate $H^* \times \Delta P$. (Again, H* is the untailed hedge ratio.) Appreciating that there is an incremental effect, we want to accrue interest on a "tailed" hedge such that Equation 6 holds.

$$H^* \times \Delta P = H \times \Delta P \times (1+r)^n$$

Equation 6

From here, Equation 7 follows.

$$H^* - T = H = H^*/(1+r)^n$$

Equation 7

Note that the calculated value for H in Equation (7) is identical to the hedge requirement H calculated from Equation (4). That is, the tailed hedge (H*-T) derived with the objective of insulating the effects of margin financing/investment to a given, deferred time is identical to the hedge needed to offset an instantaneous price effect.

II. To Tail or Not to Tail

Whichever particular market is under consideration -- whether hedging fixed-income, equity, currency exposures, or raw material price risk -- it is an open issue as to whether to tail the hedge. Because the outcome of a tailed hedge is designed to be independent of ancillary financing or investment effects connected with hedge losses or gains, it is more elegant in an economic sense; when deferral accounting is employed, however, a tailed hedge may appear to be less appealing.

With deferral accounting, gains or losses from a futures hedge are allocated to the time period that is relevant to the exposure. For example, consider the problem of hedging a variable interest expense scheduled for a future payment on June 30. With deferral accounting, hedge gains (losses) generated prior to June 30 are consolidated and deducted from (added to) actual interest expenditures paid on June 30. No adjustment is made, however, for the associated income from investing the hedge gains or financing charges on hedge losses. Rather, these incremental cash flows must be recognized during the accounting period in which they are realized.

As a consequence, tailing a hedge will necessarily foster the *appearance* of being underhedged, as the futures gains or losses realized from a tailed hedge will necessarily be smaller in magnitude than the price effect of the exposure. Certainly in some cases, this accounting concern could be overriding. When the hedge period extends into years, however, failure to tail a hedge could produce dire consequences.

A sense of the magnitude of the difference between tailed and untailed hedges can be gleaned by considering, say, a ten-year forward exposure under specific interest rate assumptions. For example, assuming a conservative discount rate of 5%, the present value factor - - $\frac{1}{(1+r)^n}$ from Equation (2) - - would be approximately 0.61, suggesting that the untailed hedge would be 39% too large. Higher (lower) interest rates would exaggerate (diminish) this difference, and of course, the degree of overhedging would be directly related to the time to the hedge value date.

A particularly well-publicized example in which hedges were not tailed is the Metallgesellschaft (henceforth MG) case. Here, a U.S. subsidiary of a German conglomerate used New York Mercantile Exchange gasoline, heating oil, and crude oil futures contracts to hedge MG's forward contract obligations with its customers. These hedges were designed to match quantities (i.e., barrels and/or gallons). That is, for each barrel/gallon sold for deferred delivery, a barrel/gallon's worth of futures contracts was purchased. The hedge design thus equated the price effects but failed to take into account the timing considerations. That is, an untailed hedge was used when a tailed hedge would have been more appropriate.[3]

The choice between tailed or untailed hedges may not have been entirely clear-cut, however, as MG′s forward contracts allowed for earlier delivery, at the customer′s discretion. This imbedded option introduces an element of uncertainty with respect to the selection of the appropriate value dates. An untailed hedge would have been appropriate if it were expected that delivery would occur imminently. A fully tailed hedge, on the other hand, would have been proper under the assumption or expectation that delivery would take place at the latest possible date allowed by the forward contract.[4]

In the specific case of MG, the company may have put itself in a box by “contractually agreeing to remain fully hedged,” presumably to cover the contingency of early exercise of this option (Culp and Miller [1995, p.64]). While it is not clear whether the cost of this hedge was fully reflected in the forward prices quoted to the customers, this pricing consideration should have been paramount in the decision to unwind or to continue the hedge in the face of mounting futures losses.

III. Tailed Spreads

In general, the decision to initiate a spread trade follows from an expectation that two typically related futures prices will move differently. When the component prices are expected to be linearly related, however, the expected price effect may be due to one of two influences: (1) a price level effect, or (2) a spread yield effect.

Treating the issue generically, consider the system of equations:

$$\begin{cases} F_1 = \alpha S \\ F_2 = \beta F_1 \end{cases}$$

Equations 8 and 9

where F_1 = the price of the first futures contract in a spread (e.g., the nearby contract in a calendar spread);

S = the underlying spot price;

F_2 = the price of the second futures contract in a spread position (e.g., the deferred contract in a calendar spread); and

α, β = coefficients of proportionality.

For storable commodities, where forward/futures prices reflect cost of carry considerations, these influences are captured in the α and β coefficients.

The spread price is thus found as follows:

$$F_2 - F_1 = \beta F_1 - \alpha S = \beta\alpha S - \alpha S = \alpha S(\beta - 1)$$

Equation 10

Simplifying:

$$F_2 - F_1 = \gamma S$$

Equation 11

Where $\gamma = \alpha(\beta-1)$.

The coefficient γ also reflects a “yield” type of consideration. That is,

$$\gamma = \frac{F_2 - F_1}{S}$$

Equation 12

The issue might best be understood by example. Assume a calendar spread involving Mexican peso futures. The coefficient of proportionality I in Equation (8) reflects the covered interest arbitrage relationship involving the interest rates in the U.S. and Mexico. The coefficient β in Equation (9) reflects the same principle, involving *forward* interest rates. A change in these underlying interest rates will thus affect the spread price.

Note, however, that a change in the underlying spot exchange rate (S) will also influence the spread price [from Equation 10), even if the contributing interest rates remain constant. It should be clear, then, that the trader who makes the trading decision based solely on yield considerations (e.g., associated interest rates, independent of exchange rates, per se) might want a position that immunizes the trade from the effects of a price level change. A tailed spread trade accomplishes this objective.

Consider, for example, an original spread position of N contracts on each of the two legs of the spread. A tail of n contracts, typically assigned to the first futures leg, is designed to offset price level effects on the N spreads, under the assumption that α, and β, and thus γ, remain constant.

$$\begin{aligned} n \times \Delta F_1 &= N \times \Delta(F_2 - F_1) \\ &= N \times \Delta(\beta F_1 - F_1) \\ &= N \times \beta \Delta F_1 - N \times \Delta F_1 \end{aligned}$$

Therefore,

$$n = N \times (\beta - 1)$$

Equation 13

Returning to Equation (9), however, note that $\beta = \frac{F_2}{F_1}$ and $(\beta - 1) = \frac{F_2}{F_1} - 1 = \frac{F_2 - F_1}{F_1}$.

Therefore, Equation (13) can be rewritten as:

$$n = N \times \left(\frac{F_2 - F_1}{F_1} \right)$$

Equation 14

To demonstrate the efficiency of the tailed spread, consider another example. Assume the nearby peso futures contract (F_1) is trading at a price of \$0.123075, and the next-out futures (F_2) is trading at \$0.118200. Given a ninety-one day interval between the two value dates, the spread yield associated with these prices is -15.67%.

Assuming a desired spread position of 100 contracts per side, this spread yield dictates a tail of four contracts.

Exhibit 2 has four sections, reflecting the consequences of varying the two futures prices (F1 on the vertical axis, and F2 shown horizontally). In the top panel (A), the spread yields are shown for all the associated pairs of futures prices. Note that the price pairs are designed so that spread yields along the diagonal (in boldface) are all equal to the initial spread yield of -15.67%. The central pair of prices reflects the starting conditions.[5] Above and to the right of this diagonal, spread yields are higher (i.e., less negative); below and to the left, spread yields are lower (more negative).

The second panel of the table (B) shows the changes in these spread yields from the -15.67% yield based on the initial futures prices, using the same price pairs as those originally shown in Panel A.

In the third panel of the table (C), the final spread prices are presented, again for the same pairs of futures prices. In this section, spread prices vary across the diagonal, becoming increasingly negative moving down and to the right.

And finally, the changes in spread prices are shown in Panel D. A comparison of the upper-left to lower-right diagonals of Panels B and D highlights the situations where spread yields are constant, but spread prices vary.

Panel A of Exhibit 3 shows the results of a 100 x 100 untailed spread - - selling the nearby and buying the deferred futures contracts -- using the same price pairs as those presented in Exhibit 2. Such a trade would be appropriate if one expected the price of the deferred futures to increase relative to the price of the nearby, which in turn could occur either because pesos were expected to strengthen relative to dollars (i.e., a price level effect) or because U.S. interest rates were expected to rise relative to Mexican interest rates.

Thus, following the imposition of this trade, the spreader would hope to move from the center-most cell (the initial position), upward to the right, where both effects work beneficially. Both effects work adversely with movement down and to the left. In the remaining two corner cells (upper-right and lower-left), the two influences are (partially) offsetting.

As would be expected, the profit and loss on the untailed spread changes directly with the spread prices, i.e., generating gains when the spread price becomes less negative. Note that the untailed spread generates non-zero results along the diagonal for all except the central location (reflecting the initial prices), even though the spread yield remains constant for these price pairs. Importantly, if the motivation for the trade were independent of a view of the peso, per se, these non-zero results would be undesirable.

Imposing a tail of four long contracts on the nearby leg of the spread along with the original 100 x 100 spread results in the profits and losses shown in Panel B of Exhibit 3.

For all intents and purposes, no gains or losses are realized along the diagonal where spread yields are identical. The small magnitudes shown simply reflect a rounding error, due to the fact that the theoretically correct tail is actually 3.97 contracts, but a slightly larger tail position is required (four contracts) because only whole numbers of contracts can be traded.

IV. Conclusion

The term "tailing" means different things when used in the context of futures hedging versus spread trading. In the first case, a tail reduces the path-dependency of hedge outcomes by mitigating the effects of variation margin financing or investing. In the second case, tailing allows the spreader to capture effects of changes in spread yields, independent of price level effects.

A prerequisite before even considering to tail is the issue of scale. That is, because of rounding considerations, smaller market participants may find tailing impractical, as the prescribed tail size may turn out to be only a fraction of a contract, and only whole numbers of futures contracts are traded. If the scale of operations is sufficient to allow for tailing, discretionary use of a tail will allow for greater control and more predictable results.

Appendix

For equity hedging situations where the funding/investment activities rely on money market instruments (i.e., where the horizon is one year or less) Equation (7) in the text can be reformatted as follows:

$$H = \frac{H^*}{\left(1 + rm\frac{d}{360}\right)}$$

Equation A1

where rm is the money market interest associated with a funding/investment horizon of d days.

Whether generated by this equation or Equation (7), the results must be identical. Note, however, that one must use notionally different but economically equivalent interest rates in the respective equations - - a bond-equivalent rate (r) in Equation (7) and a money market rate (rm) in Equation (A-1).

For example, assume these conditions:

1. An S&P500 portfolio valued at $75 million.
2. An S&P500 index at 600.00.
3. A prospective horizon of 150 days.
4. An associated bond-equivalent yield of 10% (annual compounding) or a money market rate of 9.587%.

Under such circumstances, from Equation (5):

$$H^* = \frac{75\text{million}}{600 \times 500} = 250\,\text{futures}$$

Equation A2

Solving for H from Equation (7):

$$H = \frac{250}{(1.1)^{150/365}} = 240.397$$

Equation A3

and from Equation (A1):

$$H = \frac{250}{\left(1 + .09587 \times \frac{150}{360}\right)} = 240.397$$

Equation A4

Thus, regardless of the calculation convention, the tail (i.e., H^* -H) is uniquely determined for any given yield to maturity and time horizon. Because of rounding considerations, in this example the initial tail requirement is ten contracts. Over the course of the 150-day horizon, the tail should gradually be reduced to zero.

Stated another way, the tailed hedge should increase from 240 to 250 contracts over the life of the hedge. Barring any dramatic change in interest rates, then, one would likely expect to increase this hedge position by an additional contract every fifteen days.

References

Culp, C.L., and Miller, M.H. “Metallgesellschaft and the Economics of Synthetic Storage.” Journal of Applied Corporate Finance, Winter 1995, pp. 6-21.

Edwards, F.R., and Canter, M.S. “The Collapse of Metallgesellschaft: Unhedgeable Risks, Poor Hedging Strategy, or Just Bad Luck?” The Journal of Futures Markets, May 1995, pp. 211-264.

Table 1: Equity Hedge Example

Starting Conditions	
Exposure	20,000,000
Beta	1
S&P Index	500.00
S&P Futures	502.00
Theoretical hedge ratio	80.00
Actual hedge ratio	80
Holding period	0.125
Dividend yield	2.00%
Dividend dollars	50,000
Basis adjustment ($)	80,000
Basis adjustment (%)	3.20%
Total Dollars returned	130,000
Return as MMY	5.20%
Rising Market	
Final S&P index	600.00
Final S&P futures	600.00
Capital gains	4,000,000
Futures results	(3,920,000)
Dividend results	50,000
Combined results($)	130,000
Combined results(%)	5.20%
Falling Market	
Final S&P index	400.00
Final S&P futures	400.00
Capital gains	(4,000,000)
Futures results	4,080,000
Dividend results	50,000
Combined results($)	130,000
Combined results(%)	5.20%

Table 2: Spread Yields and Prices

		F2>	0.115836	0.117018	0.118200	0.119382	0.120564
					Final Spread Yields		
		0.120614	**-15.67%**	-11.79%	-7.92%	-4.04%	-0.16%
		0.121844	-19.51%	**-15.67%**	-11.83%	-7.99%	-4.16%
(A)	F1	0.123075	-23.27%	-19.47%	**-15.67%**	-11.87%	-8.07%
		0.124306	-26.96%	-23.19%	-19.43%	**-15.67%**	-11.91%
		0.125537	-30.57%	-26.84%	-23.12%	-19.39%	**-15.67%**

		F2>	0.115836	0.117018	0.1182	0.119382	0.120564
					Change in Spread Yields		
		0.120614	**0.00%**	3.88%	7.75%	11.63%	15.51%
		0.121844	-3.84%	**0.00%**	3.84%	7.68%	11.51%
(B)	F1	0.123075	-7.60%	-3.80%	**0.00%**	3.80%	7.60%
		0.124306	-11.29%	-7.52%	-3.76%	**0.00%**	3.76%
		0.125537	-14.90%	-11.17%	-7.45%	-3.72%	**0.00%**

		F2>	0.115836	0.117018	0.118200	0.119382	0.120564
					Final Spread Prices		
		0.120614	**-0.004778**	-0.003596	-0.002414	-0.001232	-0.000049
		0.121844	-0.006008	**-0.004826**	-0.003644	-0.002462	-0.001280
(C)	F1	0.123075	-0.007239	-0.006057	**-0.004875**	-0.003693	-0.002511
		0.124306	-0.008470	-0.007288	-0.006106	**-0.004924**	-0.003742
		0.125537	-0.009701	-0.008519	-0.007337	-0.006154	**-0.004972**

		F2>	0.115836	0.117018	0.118200	0.119382	0.120564
					Change in Spread Prices		
		0.120614	**0.000098**	0.001280	0.002462	0.003644	0.004826
		0.121844	-0.001133	**0.000049**	0.001231	0.002413	0.003595
(D)	F1	0.123075	-0.002364	-0.001182	**0.000000**	0.001182	0.002364
		0.124306	-0.003595	-0.002413	-0.001231	**-0.000049**	0.001133
		0.125537	-0.004826	-0.003643	-0.002461	-0.001279	**-0.000097**

Table 3: Tailed vs. Untailed Results

Spread Size = 100 **Tail = 4**

		F2>	0.115836	0.117018	0.118200	0.119382	0.120564
			Untailed Spread Results				
		0.120614	**4,875**	63,975	123,075	182,175	241,275
		0.121844	-56,663	**2,438**	61,538	120,638	179,738
(A)	F1	0.123075	-118,200	-59,100	**0**	59,100	118,200
		0.124306	-179,738	-120,638	-61,538	**-2,438**	56,663
		0.125537	-241,275	-182,175	-123,075	-63,975	**-4,875**

		F2>	0.115836	0.117018	0.118200	0.119382	0.120564
			Tailed Spread Results				
		0.120614	**-48**	59,052	118,152	177,252	236,352
		0.121844	-59,124	**-24**	59,076	118,176	177,276
(B)	F1	0.123075	-118,200	-59,100	**0**	59,100	118,200
		0.124306	-177,276	-118,176	-59,076	**24**	59,124
		0.125537	-236,352	-177,252	-118,152	-59,052	**48**

Endnotes

[1] The equation assumes a portfolio with a beta equal to 1. Additionally, dividend distributions are considered to be exogenous and held constant.

[2] Depending on interest rate conventions, this equation may be presented in alternative formats. See the appendix for more detail on this issue.

[3] Culp and Miller [1995] and Edwards and Canter [1995] debate whether a stacked hedge (using futures contracts with nearby expirations) would approximately cover the risk. In my judgment, however, a much more critical question is whether the hedge was of the correct magnitude. That is, was the *number* of futures contracts appropriate for the risks, irrespective of the choice of expirations?

[4] Determining the appropriate horizon and therefore the appropriate tailed hedge ratio for MG's situation is a non-trivial problem. Ideally (and conceptually), this solution required a dynamic hedging process that reflects the changing deltas of the imbedded short puts, along with outright forward exposures. Such a procedure is not without risk, however, as the selection of any *specific* tail reflects an implied assumption about the delivery date of the products. If delivery occurs earlier, the hedge will be insufficient; if delivery occurs later, the hedge will be excessive.

[5] To achieve this result, we relax the pricing restriction that all peso futures must trade in quarter tick intervals (0.000025).

[*] The two rates are equivalent. Note that $(1+.09587 \times 150/360=(1.1)^{150/360}=1.039946$